CHOICE OF OPTIMUM PARTICLE DIAMETER FOR HEAT EXCHANGE IN A FLUIDIZED BED

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It was shown experimentally and by calculation that in heat transfer between an isothermal wall and a fluidized bed with all the heat removed by the fluidizing agent, the assumed transfer coefficient  $\alpha_{as}$  and the maximum flux Q vary widely with increase in particle size of fine grained materials.

One of the most important factors effecting heat transfer in a fluidized bed is the solid particle size d [1-4]. When d is made smaller than 2-5 mm the maximum heat transfer coefficient  $\alpha_{max}$  increases but simultaneously there is a decrease in the optimum mass velocity G<sub>opt</sub> of the fluidizing agent.



Fig. 1. Diagram of installation: 1) calorimeter; 2) wall; 3) movable wall; 4) insulation; 5) thermocouples; 6) caps; 7) grid; 8) fluidized agent supply.

In heat exchange with an isothermal surface when all the heat is removed (supplied) by the flow of fluidizing agent, the decrease in maximum heat flux  $Gc_p$  results in smaller temperature difference  $\theta$  between the wall and the bed.

Consequently in the relationships observed between the particle size d and heat transfer to or from the wall wide variations in the maximum heat flux Q (for given temperature of the wall and the fluidized bed at its inlet) are found. This is the case for both fluidized and packed beds independently of the bed shape.

In experiments made with the apparatus shown in Fig. 1 with the bed with coarse packing and small diameter, the temperature gradient perpendicular to the wall lies larger in a boundary layer of width comparable with the particle size. Hence when the ratio of the bed height H to the packing size s (in the experiments  $(H/s)_{max} = 13.6$ ) is large then the bed temperature varies axially with the height. Studying the uniform case, [2] the energy equation can be written

$$\frac{d^2\theta}{dz^2} - \frac{Gc_p}{\lambda_z} \frac{d\theta}{dz} - \frac{\alpha}{s\lambda_z} \theta = 0.$$
 (1)

In principle (1) is applicable for any case of uniform heat transfer distribution along the bed height and particularly for a pipe cluster in the bed if s is taken to be the ratio of free volume (not containing the heat exchanging pipes) to the heat exchange surface area. Considering that  $\alpha$  does not depend on z [4], for boundary conditions

$$z = 0 \quad \theta = \theta_0 + \frac{\lambda_z}{Gc_p} \frac{d\theta}{dz}; \quad z = H \quad \theta_{\rm B} = \theta_0 - \int_0^H \frac{\alpha\theta}{Gc_p s} dz$$

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Fig. 2. Heat transfer coefficient  $\alpha$  versus size of particles d and diameter of packing spheres  $D_s$ : 1) data of [7], glass beads 120  $\mu$ m i.d., diameter of packing spheres 19 mm; 2) d = 120  $\mu$ m;  $D_s = 19$  mm; 3) 320 and 19; 4) 320 and 44; 5) 630 and 44; 6)  $D_s = 44$  mm, d = 800  $\mu$ m; 7) 44 and 1000 respectively; 8) 28 and 1000; 9) 44 and 120; 2-9) fine-fine-grained material (corundum).  $\alpha$ , W/m<sup>2</sup>·deg; G, kg /sec · m<sup>2</sup>.

Fig. 3. Vertical thermal conductivity  $\lambda_z$  versus size of particles d and of packing  $D_s$ ; I) corundum,  $d = 320 \,\mu$ m; II)  $d = 630 \,\mu$ m; III)  $d = 1000 \,\mu$ m. I-III)  $D_s = 44 \,\mu$ m; IV)  $d = 1000 \,\mu$ m;  $D_s = 28 \,\mu$ m. I-IV)  $s = 0.11 \,\mu$ ;  $H = 1.4 \,\mu$ .  $\lambda$ , W/m·deg.

the solution of Eq. (1) can be rewritten

$$\frac{\theta}{\theta_0} = A e^{\gamma_1 z} + B e^{\gamma_2 z},\tag{2}$$

where

$$A = \frac{Gc_p}{\lambda_z \gamma_2} - B \frac{\gamma_1}{\gamma_2}; \quad B = \frac{e^{\gamma_1 H} \left(\frac{Gc_p}{\lambda_z \gamma_2} - 1\right)}{\frac{\gamma_1}{\gamma_2} e^{\gamma_1 H} \pm \frac{\alpha}{Gc_p s \gamma_2} (e^{\gamma_2 H} - e^{\gamma_2 H}) - e^{\gamma_2 H}};$$
(3)  
$$\gamma_1 = \frac{1}{2} \frac{Gc_p}{\lambda_z} - \sqrt{\left(\frac{1}{2} \frac{Gc_p}{\lambda_z}\right)^2 \pm \frac{\alpha}{s\lambda_z}};$$
$$\gamma_2 = \frac{1}{2} \frac{Gc_p}{\lambda_z} + \sqrt{\left(\frac{1}{2} \frac{Gc_p}{\lambda_z}\right)^2 + \frac{\alpha}{s\lambda_z}}.$$
(4)

The maximum heat flux

$$Q = \int_{0}^{H} \alpha \theta l dz = \theta_{\theta} \alpha l \left[ \frac{A}{\gamma_{1}} \left( e^{\gamma_{1} H} - 1 \right) + \frac{B}{\gamma_{2}} \left( e^{\gamma_{2} H} - 1 \right) \right].$$
(5)

The value characterizing the heat flux for unit surface area related to the temperature difference between the wall and the fluidizing agent at entry has the dimension of the heat transfer coefficient and is given by

$$\alpha_{\rm con} = \frac{Q}{F\theta_0} = \frac{\alpha}{H} \left[ \frac{A}{\gamma_1} \left( e^{\gamma_1 H} - 1 \right) + \frac{B}{\gamma_2} \left( e^{\gamma_2 H} - 1 \right) \right]. \tag{6}$$

Changing to dimensionless form

$$\frac{\lambda_z}{HGc_p} = \frac{1}{\text{Pe}} \equiv a; \quad \frac{\alpha H}{Gc_p s} = b; \quad \frac{\alpha_{\text{con}} H}{Gc_p s} = \psi.$$



Fig. 4. Conventional heat transfer coefficient  $\alpha_{as}$  (W/m<sup>2</sup>·deg) versus dimensions of installation, diameters of packing spheres D<sub>s</sub> and of particles d, mm. Roman numerals  $\lambda_z = \infty$ , Arabic numerals  $\lambda_z = 0$ , letters, experimental and those calculated by (13) values of points, authors' experimental data. 1, A, I) D<sub>s</sub> = 44 mm; H = 1.4 m; s = 0.11 m; 2, B, II) D<sub>s</sub> = 44 mm; H = 0.56 m; s = 0.13 m; 3, III) D<sub>s</sub> = 28 mm; H = 0.56 m; s = 0.13 m; 4, C, IV) free layer without regard for  $\alpha_{con}$ ; s = 0.2 m; H = 0.4 m; V) the same; s = 0.15 m; H = 0.4 m; VI and VII) with regard for  $\alpha_{con}$ ; H = 0.4 m; s = 0.2 and and 0.3 m respectively.

Fig. 5.  $\alpha_{as}$  versus particle size d and ratio H/D<sub>a</sub> (with no regard for  $\alpha_{con}$ ;  $\lambda_z = \infty$ , free layer). 1-6) particle size 0.12; 0.32; 0.5; 0.63; 0.8 and 1.0 mm, respectively.

Equation (6) can be rewritten as

$$\psi = 1 - \left[ 1 + \frac{ab[-] \operatorname{sh} \sqrt{-}}{[+] \operatorname{ch} \sqrt{-} + ab \operatorname{sh} \sqrt{-}} \right] \exp \frac{1}{a} [-],$$
(7)

where

$$V = \sqrt{0.25 + ab}$$
;  $[-] = 0.5 - V$ ;  $[+] = 0.5 + V$ .

In many cases Eqs. (2) and (6) can be simplified and take the form:

when a = 0 ( $\lambda_z = 0$ , piston flow)

$$\theta = \theta_0 e^{-\frac{bz}{H}}; \quad \psi = 1 - e^{-b} , \qquad (8)$$

when  $a = \infty$  ( $\lambda_z = \infty$ , complete mixing)

$$\theta = \theta_0 \frac{1}{b+1}; \quad \psi = \frac{b}{b+1}. \tag{9}$$

An experimental investigation of the variation in  $\alpha(\alpha_{as})$  with d was made with the apparatus shown in Fig. 1. The bed packing consisted of graded metal spheres of diameters from 19 to 44 mm, and for fine grained material various fractions of corundum particles from 120 to 1600  $\mu$ m size, s varied from 110 to 310 mm, the height of the covered packing varied from 560 to 1400 mm. The temperature of the wall was measured with a chromel-alumel thermocouple. Thermocouples with enclosed junctions were used to measure the bed temperature, and finally six thermocouples were calked into the metal spheres. Fluidization was achieved by undried air. If the experimental graph of the change of the temperature with height of the bed is plotted in semilogarithmic form, then from the slope of the straight line  $\ln \theta / \theta_0$ —z which is only negligibly affected by the second term in Eq. (2) the value of  $\gamma_1$  can be found as the tangent of this slope. Hence

$$\lambda_z = \frac{\frac{\alpha}{s} + Gc_p \gamma_1}{\gamma_1^2}.$$
 (10)

The heat transfer coefficient is calculated from

$$\alpha = \frac{N}{f\theta} , \qquad (11)$$

Part of the experimental data for the variation of  $\alpha$  and  $\lambda_z$  with the mass velocity G of fluidizing bed agent, the size d of the particles and the packing  $D_s$  are given in Figs. 2 and 3.

From these data for  $\alpha_{\max}$ ,  $G_{opt}$ , and  $\lambda_z$  (and also when  $\lambda_z = 0$  and  $\lambda_z = \infty$ ) calculations were made of  $\alpha_s$  for various combinations of  $D_a$ ; H; d and  $D_s$  (Fig. 4). The calculation of  $\alpha_{as}$  for the free layer (Fig. 4) was made using the data [2-5];

$$\alpha_{\rm max} = 35.7 \,\rho_{\rm M}^{0.2} \,\lambda_{\rm c}^{0.6} d^{-0.36}; \tag{12}$$

$$G_{\rm opt} = \frac{\mu \,{\rm Ar}}{d \,(18 + 5,22 \,\sqrt{\rm Ar})};$$
 (13)

$$\frac{k}{v} = |34.2 D_a| (1-\varepsilon)^{-1} (\omega - 1)^{0.54} \operatorname{Ar}^{0.144} (H/D_a)^{1.3};$$
(14)

$$G_{\rm rc} = \frac{\mu \,{\rm Ar}}{d\,(1400 + 5.22 \,\mu \,\,\overline{\rm Ar})} \,\,; \tag{15}$$

$$Nu_{con} = 0.009 \,\mathrm{Ar}^{0.5} \,\mathrm{Pr}^{0.33}. \tag{16}$$

Since (12) is empirical, calculation for coarse particles was made in two different ways: in the first case it was assumed that it gives the whole of the maximum heat transfer coefficient and in the second case (curves VI and VII Fig. 4)  $\alpha_{\rm CON}$  was added to  $\alpha$  from calculation using Eq. (16). The calculated and experimental data in Fig. 4 show that for all the apparatus size and heat transfer surfaces chosen, the variation of  $\alpha_{\rm as}$  with particle size d is non-uniform. Comparison of curves IV and VI in Fig. 4 shows that addition of the convection term increases  $d_{\rm opt}$  into the range of large values for d. The experimental variation of  $\alpha_{\rm as} = f(d)$  was noted with and without the added convective term. Increasing the bed diameter (or more generally the rates of the fluidized region to the heat transfer area) allows for the same condition a large mass velocity of fluidizing agent and consequently  $d_{\rm opt}$  is decreased and  $\alpha_{\rm as}$  is increased.

This applies for the specified ranges of variables but beyond these (particularly with the packing) a limitation in the transfer horizontally is observed in the bed and consequently the present method of calculation ceases to be applicable.

With decrease in the packing size the mixing of fine particles deteriorates and there is an increase in the bed diameter [7] which decreases  $\alpha_{max}$  and therefore also  $\alpha_{as}$ .

Thus the optimum d value at which  $\alpha_{as}$  is a maximum depends on the physical properties of the fluidizing agent, the bed dimensions, the heat transfer surface area, and with a packed bed on the geometric parameters of the packing. Equations are given allowing the calculation of the heat flux Q for given bed dimensions and temperature difference  $\theta_0$  between the wall and the gas inlet or the determination of the bed dimensions for given Q and  $\theta_0$ . In packed beds the optimum d value can be found by two methods of calculation. For free beds the dopt can be chosen from Fig. 5.

## NOTATION

Q	is tl	he heat	flux,W;
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- N is the calorimeter power, W;
- $\alpha_{as}$  is the conventional heat transfer coefficient,  $W/m^2 \cdot deg$ ;
- $c_{p}$  is the heat capacity of fluidizing agent,  $J/kg \cdot deg$ ;
- **s** is the transverse dimension of installation, m;
- D<sub>a</sub> is the diameter of apparatus, m;

H	is the height of heat transfer surface, m;
d	is the diameter of fine-granular material particles, m;
l	is the width of heat transfer surface, m;
k	is the thermal diffusivity, $m^2/s$ ;
f	is the calorimeter area, m <sup>2</sup> ;
F	is the total heat transfer surface, $m^2$ ;
G	is the mass velocity of fluidizing agent, $kg/sec \cdot m^2$ ;
Gont	is the optimum mass velocity, $kg/sec \cdot m^2$ ;
Gre	is the rate of fluidization onset, $kg/sec \cdot m^2$ ;
W	is the fluidization number, $kg/sec \cdot m^2$ ;
3	is the bed porosity;
q	is the free fall acceleration, $m/sec^2$ ;
$\lambda_{c}$	is the air thermal conductivity, $W/m \cdot deg$ ;
$\lambda_z$	is the effective vertical thermal conductivity of bed, W/m ·deg;
ν	is the kinematic viscosity of air, $m^2 \cdot sec$ ;
$\mu$	is the dynamic viscosity of air, N $/m^2$ ;
$\rho_{M}$	is the density of fine-granular material, $kg/m^3$ ;
$\rho_{\rm B}$	is the air density, $kg/m^3$ ;
θ	is the temperature difference between isothermal wall and bed, $^{\circ}C$ ;
$\gamma_1, \gamma_2$	are the coefficients, 1/m;
A, B, a, b, ψ	are the dimensionless coefficients;
Ar	is the Archimedean number;
Nu	is the Nusselt number;
Pr	is the Prandtl number.

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